Literature Review

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1 Introduction

Since mankind first climbed down from the trees, it is our ability to communicate that has made us unique. Once ideas could be passed from person to person, it made sense to have a permanent record of them; one which could be passed on from person to person without them ever meeting.

And thus the document was born.

Traditionally, documents have been static: just marks on paper, but with the advent of computers many more possibilities open up.

2 Document Formats

Most existing document formats — such as the venerable PostScript and PDF — are, however, designed to imitate existing paper documents, largely to allow for easy printing. In order to truly take advantage of the possibilities operating in the digital domain opens up to us, we must look to new formats.

Formats such as `HTML` allow for a greater scope of interactivity and for a more data-driven model, allowing the content of the document to be explored in ways that perhaps the author had not anticipated.[1] However, these data-driven formats typically do not support fixed layouts, and the display differs from renderer to renderer.

2.1 A Taxonomy of Document formats

The process of creating and displaying a document is a rather universal one (2.1), though different document formats approach it slightly differently. A document often begins as raw content: text and images (be they raster or vector) and it must end up as a set of photons flying towards the reader's eyes.
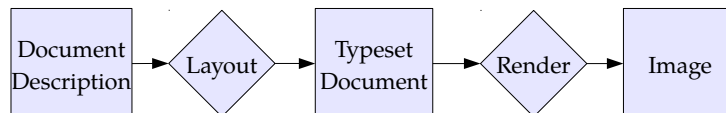

Figure 1: The lifecycle of a document

There are two fundamental stages by which all documents — digital or otherwise — are produced and displayed: *layout* and *rendering*. The *layout* stage is where the positions and sizes of text and other graphics are determined. The text will be *flowed* around graphics, the positions of individual glyphs will be placed, ensuring that there is no undesired overlap and that everything will fit on the page or screen.

The *display* stage actually produces the final output, whether as ink on paper or pixels on a computer monitor. Each graphical element is rasterized and composited into a single image of the target resolution.

Different document formats cover documents in different stages of this project. Bitmapped images, for example, would represent the output of the final stage of the process, whereas markup languages typically specify a document which has not yet been processed, ready for the layout stage.

Furthermore, some document formats treat the document as a program, written in a (usually turing complete) document language with instructions which emit shapes to be displayed. These shapes are either displayed immediately, as in PostScript, or stored in another file, such as with TeX or LaTeX, which emit a `DVI` file. Most other forms of document use a *Document Object Model*, being a list or tree of objects to be rendered. `DVI`, `PDF`, `HTML`[1] and SVG[2]. Of these, only `HTML` and TeX typically store documents in pre-layout stages, whereas even turing complete document formats such as PostScript typically encode documents which already have their elements placed.

TeX and LaTeX Donald Knuth's typesetting language TeX is one of the older computer typesetting systems, originally conceived in 1977[3]. It implements a turing-complete language and is human-readable and writable, and is still popular due to its excellent support for typesetting mathematics. TeX only implements the "layout" stage of document display, and produces a typeset file, traditionally in `DVI` format, though modern implementations will often target `PDF` instead.

This document was prepared in LaTeX 2_ε.

DVI `DVI` is a binary format which consists of a simple stack machine with instructions for drawing glyphs and curves[4].

Existing document formats, due to being designed to model paper, have limited precision (8 decimal digits for PostScript[5], 5 decimal digits for PDF[6]). This matches the limited resolution of printers and ink, but is limited when compared to what aught to be possible with "zoom" functionality, which is prevent from working beyond a limited scale factor, lest artefacts appear due to issues with numeric precision.

3   Rendering

Computer graphics comes in two forms: bit-mapped (or raster) graphics, which is defined by an array of pixel colours; and *vector* graphics, defined by mathematical descriptions of objects. Bit-mapped graphics are well suited to photographs and are match how cameras, printers and monitors work. However,

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[1]Some of these formats — most notably `HTML` — implement a scripting lanuage such as JavaScript, which permit the DOM to be modified while the document is being viewed.

bitmap devices do not handle zooming beyond their "native" resolution — the resolution where one document pixel maps to one display pixel —, exhibiting an artefact called pixelation where the pixel structure becomes evident. Attempts to use interpolation to hide this effect are never entirely successful, and sharp edges, such as those found in text and diagrams, are particularly affected.

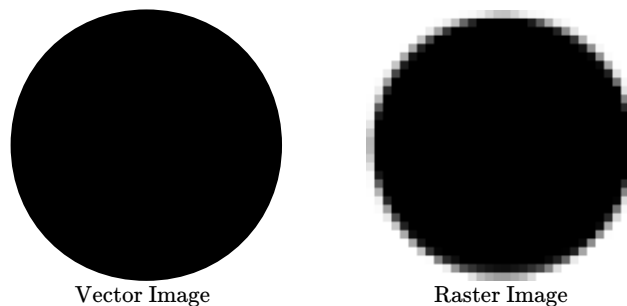<div align="center">Vector Image   Raster Image</div>

Figure 2: A circle as a vector image and a 32×32 pixel raster image

Vector graphics lack many of these problems: the representation is independent of the output resolution, and rather an abstract description of what it is being rendered, typically as a combination of simple geometric shapes like lines, arcs and "Bézier curves"[7]. As existing displays (and printers) are bit-mapped devices, vector documents must be *rasterized* into a bitmap at a given resolution. This bitmap is then displayed or printed. The resulting bitmap is then an approximation of the vector image at that resolution.

This project will be based around vector graphics, as these properties make it more suited to experimenting with zoom quality.

The rasterization process typically operates on an individual "object" or "shape" at a time: there are special algorithms for rendering lines[8], triangles[9], polygons[10] and Bézier Curves[11]. Typically, these are rasterized independently and composited in the bitmap domain using Porter-Duff compositing[12] into a single image. This allows complex images to be formed from many simple pieces, as well as allowing for layered translucent objects, which would otherwise require the solution of some very complex constructive geometry problems.

While traditionally, rasterization was done entirely in software, modern computers and mobile devices have hardware support for rasterizing some basic primitives — typically lines and triangles —, designed for use rendering 3D scenes. This hardware is usually programmed with an API like `OpenGL`[13].

More complex shapes like Bézier curves can be rendered by combining the use of bitmapped textures (possibly using signed-distance fields[14][15][16]) with polygons approximating the curve's shape[17][18].

Indeed, there are several implementations of entire vector graphics systems using OpenGL: OpenVG[19] on top of OpenGL ES[20]; the Cairo[21] library, based around the PostScript/PDF rendering model, has the "Glitz" OpenGL backend[22] and the SVG/PostScript GPU renderer by nVidia[23] as an OpenGL extension[24].

4 Numeric formats

On modern computer architectures, there are two basic number formats supported: fixed-width integers and *floating-point* numbers. Typically, computers natively support integers of up to 64 bits, capable of representing all integers between 0 and $2^{64} - 1$, inclusive[2].

By introducing a fractional component (analogous to a decimal point), we can convert integers to *fixed-point* numbers, which have a more limited range, but a fixed, greater precision. For example, a number in 4.4 fixed-point format would have four bits representing the integer component, and four bits representing the fractional component:

$$\underbrace{0101}_{\text{integer component}} \quad . \quad \underbrace{1100}_{\text{fractional component}} \quad = 5.75 \tag{1}$$

Floating-point numbers[25] are the binary equivalent of scientific notation: each number consisting of an exponent (e) and a mantissa (m) such that a number is given by

$$n = 2^e \times m \tag{2}$$

The IEEE 754 standard[26] defines several floating-point data types which are used[3] by most computer systems. The standard defines 32-bit (8-bit exponent, 23-bit mantissa, 1 sign bit) and 64-bit (11-bit exponent, 53-bit mantissa, 1 sign bit) formats[4], which can store approximately 7 and 15 decimal digits of precision respectively.

Floating-point numbers behave quite differently to integers or fixed-point numbers, as the representable numbers are not evenly distributed. Large numbers are stored to a lesser precision than numbers close to zero. This can present problems in documents when zooming in on objects far from the origin.

IEEE floating-point has some interesting features as well, including values for negative zero, positive and negative infinity, the "Not a Number" (NaN) value and *denormal* values, which trade precision for range when dealing with very small numbers. Indeed, with these values, IEEE 754 floating-point equality does not form an equivalence relation, which can cause issues when not considered carefully.[28]

There also exist formats for storing numbers with arbitrary precising and/or range. Some programming languages support "big integer"[29] types which can represent any integer that can fit in the system's memory. Similarly, there are arbitrary-precision floating-point data types[30][31] which can represent any number of the form

$$\frac{n}{2^d} \quad n, d \in \mathbb{Z} \tag{3}$$

These types are typically built from several native data types such as integers and floats, paired with custom routines implementing arithmetic primitives.[32] These, therefore, are likely slower than the native types they are built on.

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[2]Most machines also support *signed* integers, which have the same cardinality as their *unsigned* counterparts, but which represent integers in the range $[-(2^{63}), 2^{63} - 1]$

[3]Many systems' implement the IEEE 754 standard's storage formats, but do not implement arithmetic operations in accordance with this standard.

[4]The 2008 revision to this standard[27] adds some additional formats, but is less widely supported in hardware.

While traditionally, GPUs have supported some approximation of IEEE 754's 32-bit floats, modern graphics processors also support 16-bit[33] and 64-bit[34] IEEE floats. Note, however, that some parts of the GPU are only able to use some formats, so precision will likely be truncated at some point before display. Higher precision numeric types can be implemented or used on the GPU, but are slow.[35]

Pairs of integers ($a \in \mathbb{Z}, b \in \mathbb{Z} \setminus 0$) can be used to represent rationals. This allows values such as $\frac{1}{3}$ to be represented exactly, whereas in fixed or floating-point formats, this would have a recurring representation:

$$\underbrace{0}_{\text{integer part}} \quad . \quad \underbrace{01}_{\text{recurring part}} \quad 01 \quad 01 \quad 01 \ldots \tag{4}$$

Whereas with a rational type, this is simply $\frac{1}{3}$. Rationals do not have a unique representation for each value, typically the reduced fraction is used as a characteristic element.

5 Quadtrees

When viewing or processing a small part of a large document, it may be helpful to only processs — or *cull* — parts of the document which are not on-screen.
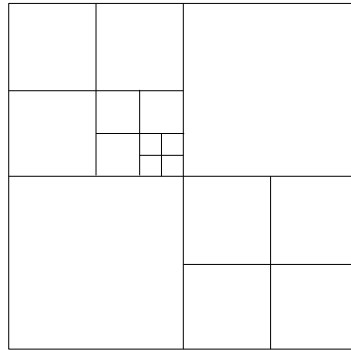

Figure 3: A simple quadtree.

The quadtree[36]is a data structure — one of a family of *spatial* data structures — which recursively breaks down space into smaller subregions which can be processed independently. Points (or other objects) are added to a single node, which if certain criteria are met — typically the number of points in a node exceeding a maximum, though in our case likely the level of precision required exceeding that supported by the data type in use — is split into four equal-sized subregions, and points attached to the region which contains them.

In this project, we will be experimenting with a form of quadtree in which each node has its own independent coordinate system, allowing us to store some spatial information[5] within the quadtree structure, eliminating redundancy in the coordinates of nearby objects.

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[5]One bit per-coordinate, per-level of the quadtree

Other spatial data structures exist, such as the KD-tree[37], which partitions the space on any axis-aligned line; or the BSP tree[38], which splits along an arbitrary line which need not be axis aligned. We believe, however, that the simpler conversion from binary coordinates to the quadtree's binary split make it a better avenue for initial research to explore.

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