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Number Representations and Precision in Vector Graphics

Implementation of an Arbitrary Precision SVG Viewer

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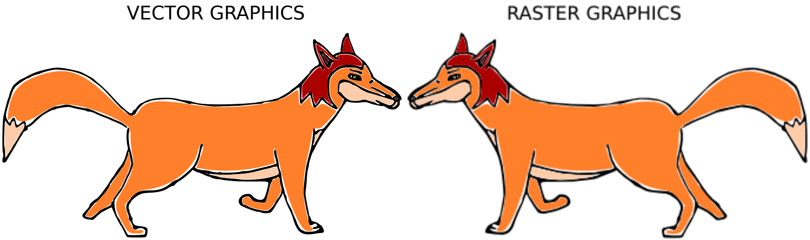
Summary

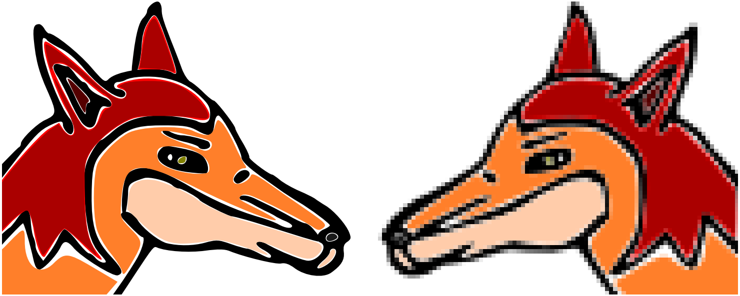
- Vector graphics allow detail to be scaled but not by an arbitrary amount
- We've implemented a vector graphics viewer which does allow arbitrary scaling

Graphics Formats

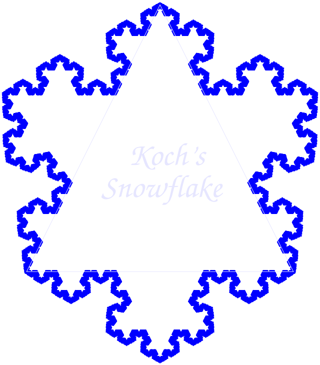
- Document formats (eg: PDF and SVG) are formats for vector graphics
- Vector graphics scale better than raster graphics



- Vector graphics scale better than raster graphics



Is there a zoom limit?



Is there a zoom limit?

- SVG, PostScript, PDF specify IEEE-754 *single* floating point number representations
- Range of values: $\approx 3 \times 10^{-38} \rightarrow 3 \times 10^{+38}$
- Bigger than size of Universe?

Floating Point Definition

- Rough Floating Point Definition[1]:

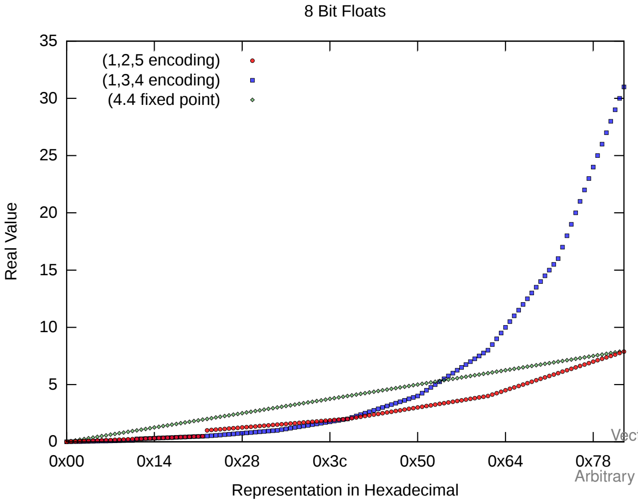
$$X = m \times 2^E \tag{1}$$

- m and E are encoded in a *fixed length* string of bits
- Floating Point $\approx$ Scientific Notation for computers

[1] IEEE-754 is more complicated

Visualisation of Floats

- With total length of *m* and *E* limited to 7 bits (1 sign bit)
- Operations are inexact (in general)



Visualisation of Floats II

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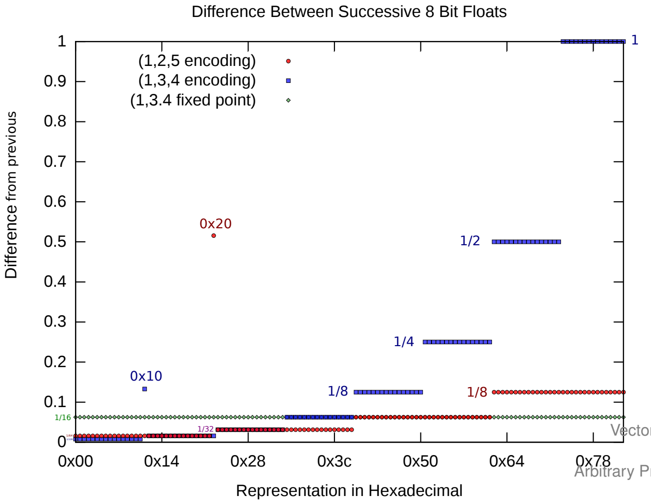
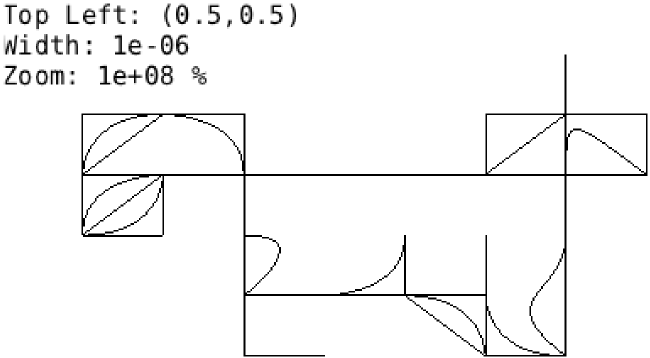
- Difference between successive floats
- Further from origin $\Longrightarrow$ less precision



Motivation
Floating Point
Vector Graphics Viewer
Precision Issues
Arbitrary Precision Rationals
Demonstration
Conclusions

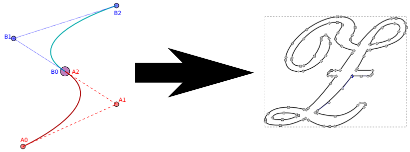
Precision is limited

- Eg: Fox scaled to width of $\approx 10^{-6}$ viewed at zoom of $\approx 10^{8}$
- (Outline should look like images in first slide)



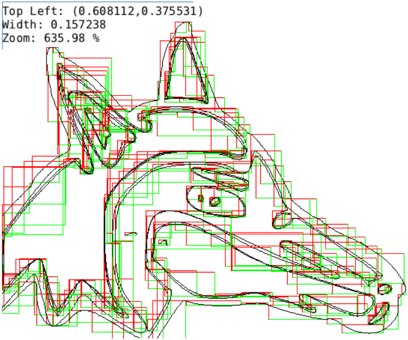
Structure of Vector Graphics

- Bézier Curve (Quadratic or Cubic Parametric Polynomial)
- Path of Bézier Curves $\rightarrow$ Shapes (with fill)
- Shapes include font glyphs, like this $\mathscr{L}$



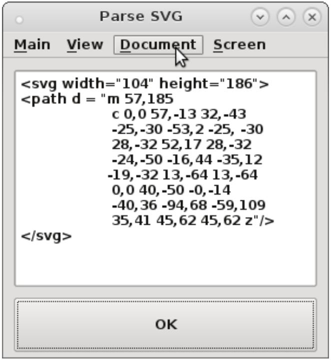
Structure of Vector Graphics II

- Upload *bounding rectangles* of individual objects to renderer (OpenGL)
- Rectangles show individual Béziers forming outline of the Fox



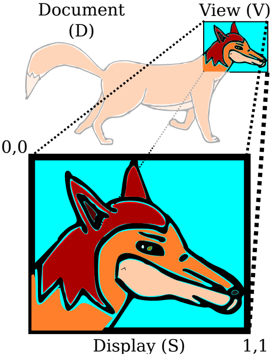
Vector Graphics Viewer Features

- GPU (OpenGL) or CPU (custom) rendering
- Import SVG into current View location
- Control through scripts (Python) or Qt4 GUI



Viewing Vector Graphics

- Tranform coordinates in document → display



Document via View to Display (S)

$$(X, Y) \overset{V}{\longmapsto} (S_X, S_Y)$$

$$S_X = \frac{X - V_x}{V_w} \quad S_Y = \frac{Y - V_y}{V_h}$$

Floating point calculations go wrong

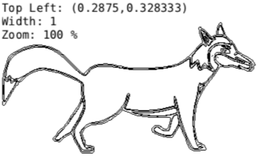
- Example: Insert objects at very small scale

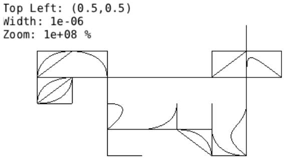
$$X = V_w \times O_x + V_x$$

Round to floats near Vx

$$S_X = \frac{X - V_x}{V_w}$$

Increases error in X





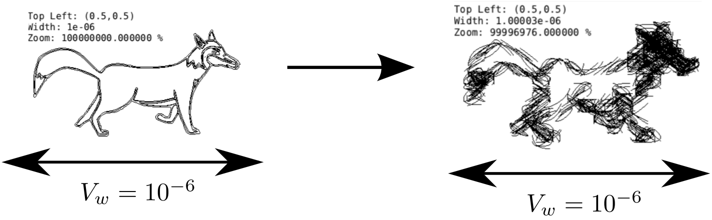
$V_w = 1$ ✓

$V_w = 10^{-6}$ ✗

Reducing error

- Don't apply view transformation directly
- Store object bounds relative to the display
- When modifying the view, modify object bounds
- Detail inserted into the view looks good, But...

Zoom out by a large amount then back in



Inserted SVG

Accumulated Errors

Cumulated Errors with intermediate coord system

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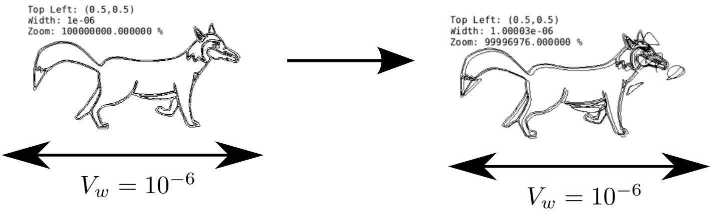
- Apply transformations to Paths not individual Béziers
- Paths render correctly, but drift appart

Zoom out by a large amount then back in



Inserted SVG

Drifting of Paths

We only need to transform the Paths with increased precision

Arbitrary Precision Rationals

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$$Q = \frac{N}{D} \tag{2}$$

- N and D are arbitrary precision integers

$$N = \sum_{i=0}^{S} d_i \beta^i \tag{3}$$

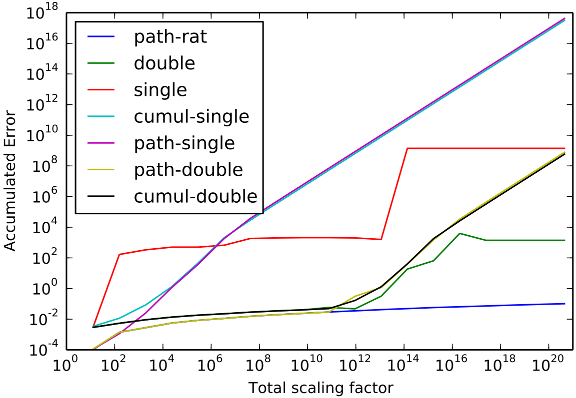
- d_i are fixed size integers, $\beta = 2^{64}$
- **Size S grows as needed**
- Operations are always exact
- Implemented by GNU Multiple Precision Library

Use Rationals to represent Path Coordinates

- Can move view to arbitrary point
- Insert detail (ie: Test SVG image) in Display coordinates
- Move view to another arbitrary point
- Move view back
- Detail is unchanged

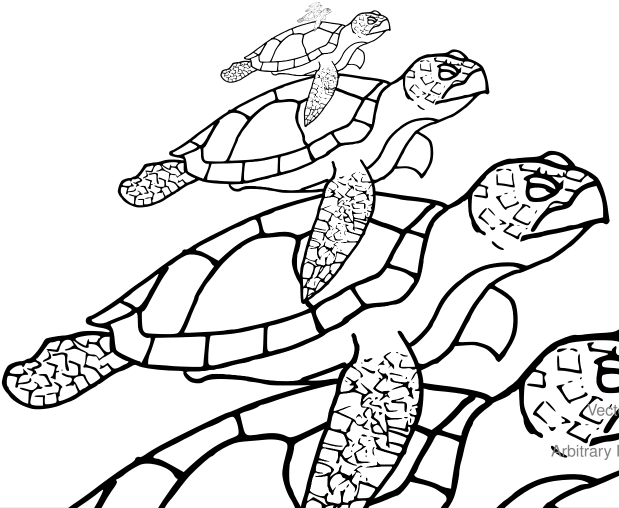
Quantitative Results

- Invariance of grid of lines after scaling



Demonstration

- We can import standard SVGs wherever we want
- If we are willing to wait long enough
- “... But, asks the scientist, what does that turtle stand on? To which the lady triumphantly answers: ’You’re very clever, young man, but it’s no use – it’s turtles all the way down!’.“



What was done

- Implemented a basic SVG viewer
- Demonstrated how precision affects rendering vector graphics
- Using GMP rationals, demonstrated the ability to render SVGs scaled to an arbitrary position in a document

Future work

- Implement more of the SVG standard
- Trial alternative number representations
- Allow for saving and loading
- Optimisations, eg: clip objects that are not visible
- Compile for Windows (MinGW)

Acknowledgements

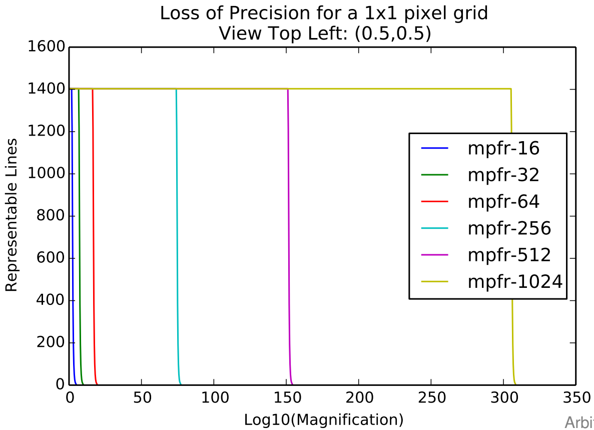
- Work on SVG viewer collaborative with David Gow
  - See David Gow's presentation about Quadtrees
- Supervisors: Tim French and Rowan Davies

References & More information

- Muller et al, *Handbook of Floating Point Arithmetic*,
- Hearn, Baker *Computer Graphics*
- Kahan et al, *IEEE-754* (1985 and 2008 revision)
- Dahlstóm et al, *SVG WC3 Recommendation 2011*
- Grunland et al, *GNU Multiple Precision Manual 6.0.0a*
- Kahan's website
  `http://http.cs.berkeley.edu/~wkahan`

Q: Why not just increase float precision?

- GPU uses singles anyway
- Can use CPU for bounds transforms
- But eventually lose precision for any *fixed* precision float



Q: Arbitrary precision floats?

$$X = m \times 2^E \quad (4)$$

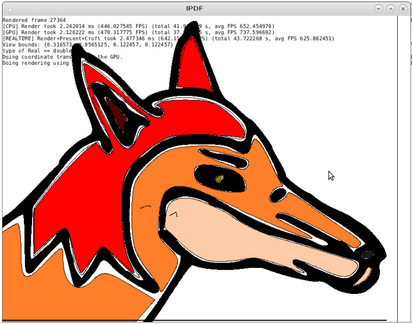
- m and E are of arbitrary size
- Implemented by MPFR (based on GMP)
- Difficulties:
  - Need to manually set precision (size) of m
  - Some operations require infinite precision:

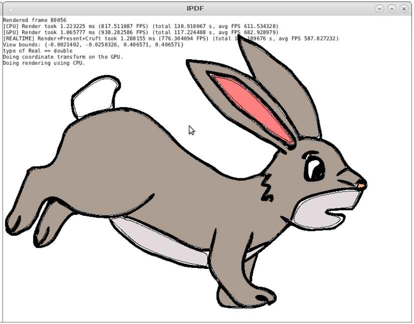
$$\frac{1}{3} = 0.33333333333333333333333\ldots \times 10^0 \quad (5)$$

  - How do you choose when to increase precision?
  - GMP Rational implementation automatically increases size, but MPFR floats do not

Q: Why don't you have colour?

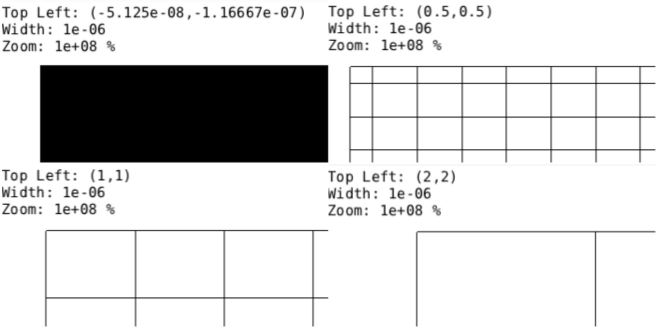
- We do![2]
- A complete implementation of SVG is "future work"



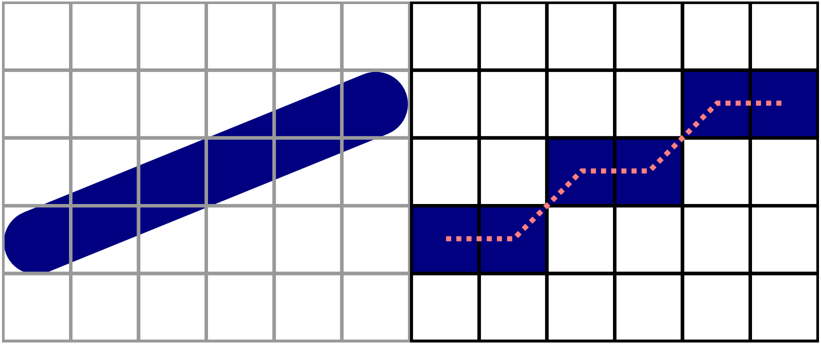


[2] If you are willing to wait long enough

Quantitative?



Bresenham and Wu



Bonus: IEEE-754 on GPUs

- Inconsistent behaviour of calculations on different GPUs
- Eg: $x^2 + y^2 < 1$ (shading a circle) zoomed in on the edge.